simply supported (SS) boundary conditions are considered: $u_0 = w_0 = \psi_y = \phi_y = 0$ at $x = 0$, a and $v_0 = w_0 = \psi_x = \phi_x = 0$ at $y = 0$, b.

The thermoelastic material properties of P55/6051 graphite/aluminum (Gr/Al) considered are: E_1 = 190.0 GPa, E_2 = 48.3 GPa, $G_{12} = G_{13}$ = 17.3 GPa, G_{23} = 16.5 GPa, ν_{12} = 0.28, ρ = 2400.0 kg/m^3, $\alpha_1 = 3.34 \times 10^{-6}$ m/m/°C, $\alpha_2 = 26.1 \times 10^{-6}$ m/m/°C.

Unless mentioned otherwise, a square plate of length 254 mm with a side-to-thickness ratio of 50 is considered for the analysis. The values of T_1 and T_2 are taken as 250 and 500°C, respectively. A time step of 4×10^{-5} s is used for all of the problems considered. Figure 1 shows the linear and nonlinear transient responses of a two-layer cross-ply SS plate. The effect of nonlinearity is to decrease the amplitude and period of the center deflection. Figure 2 shows the effect of the ply orientation on the nonlinear transient response of a SS plate. Of the three lamination schemes considered, the behavior of [0/90 deg] and [45/−45 deg] laminates are found to be similar. Figure 3 shows the effect of side-to-thickness ratio on the nonlinear response of a two-layer cross-ply SS plate. The effect of the thickness on the amplitude of the center deflection is apparent from the figures.

Conclusions

A nonlinear higher order shear deformation theory is used for the dynamic analysis of laminated composite plates. A C^0 continuous finite element model is developed. In contrast to the first-order shear deformation theory, the present higher order theory does not require shear correction factor due to the more realistic representation of the cross-sectional deformation. Numerical results are presented for metal matrix based composite plates which should serve as benchmarks for future studies.

References

[1]Tauchert, T. R., "Thermally Induced Flexure, Buckling, and Vibration of Plates," *Applied Mechanics Review*, Vol. 44, No. 8, 1991, pp. 347–360.

[2]Reddy, J. N., and Hsu, Y. S., "Effects of Shear Deformation and Anisotropy on the Thermal Bending of Layered Composite Plates," *Journal of Thermal Stresses*, Vol. 3, No. 4, 1980, pp. 475–493.

[3]Tauchert, T. R., "Thermal Buckling of Thick Antisymmetric Angle-Ply Laminates," *Journal of Thermal Stresses*, Vol. 10, No. 2, 1987, pp. 113–124.

[4]Chandrashekhara, K., "Thermal Buckling of Laminated Plates Using a Shear Flexible Finite Element," *Finite Elements in Analysis and Design*, Vol. 12, No. 1, 1992, pp. 51–61.

[5]Reddy, J. N., "A Simple Higher-Order Theory for Laminated Composite Plates," *Journal of Applied Mechanics*, Vol. 51, No. 4, 1984, pp. 745–752.

Hierarchy in the Design and Development of Structural Components: A Preliminary Study

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Introduction

STRUCTURAL components designed and built on the principle of hierarchy have the potential to develop extraordinary capabilities with regard to strength and toughness. Biological structures such as wood, nacre, bone, and cuticle illustrate the potentials of hierarchy extending over several scales in their "design." In this paper an attempt to model simple ropelike structures in tension is discussed from the point of view of examining their suitability for use in composites. Numerical results from a parametric study of a single strand in tension indicate that although an increase in toughness arises due to increased ductility of the element by virtue of its architecture, firm conclusions cannot be drawn until the response in compression is evaluated.

Hierarchy, in the context of this paper, is defined as an arrangement of material and/or structure in which a basic element repeats itself over several increasing scales leading to the desired structural configuration. There is experimental evidence to show that composite panels simulating the hierarchical architecture of wood exhibit extraordinary structural integrity that the material/structure would lack otherwise. The resistance of these panels to impact loads was demonstrated to be far superior to equivalent aluminum panels.[1] A result of the material/structural architecture of hierarchy is that any induced damage is resisted by a series of interruptions in the progress of such damage leading to a tortuous crack path. It is this basic observation that influences consideration of hierarchical elements for structural components.

Ropelike structures are, perhaps, the oldest structural components that have withstood the tests of time in mining, marine, and elevator industries. Our interest is to examine such elements for use as fibers in advanced composites for a wide variety of uses where impact loads may be particularly significant. This paper is an attempt at a preliminary examination of the structural characteristics of such strands. This study does not include the important consideration of the fiber matrix interface that the inherent undulations in the filter architecture are ultimately likely to offer a better interfacial bonding. The use of multimaterial helically wound fibers is expected to add significantly to the design space for new materials. Although use of ropelike fibers with different materials for the core and outer layers is not a new concept, the authors be-

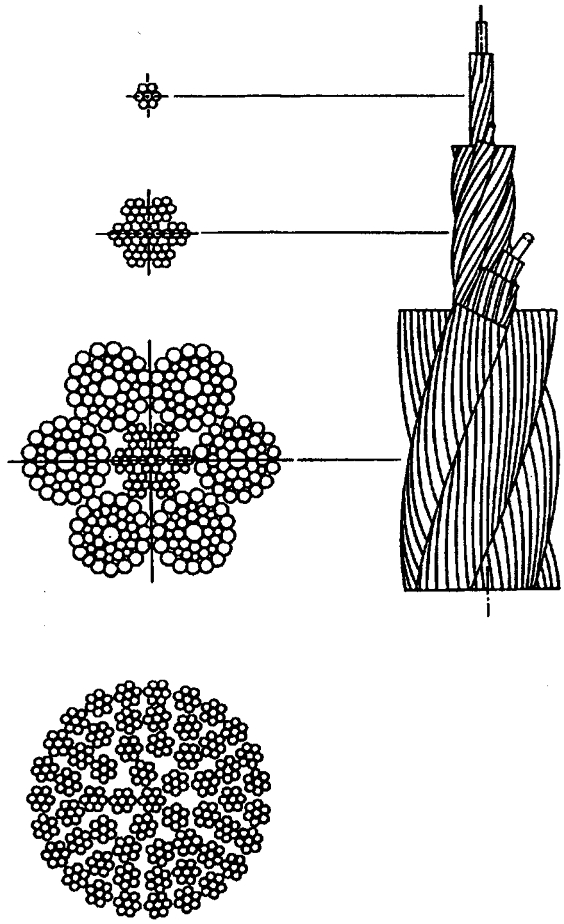

Fig. 1 Illustrative rope constructions (Lee[3]).

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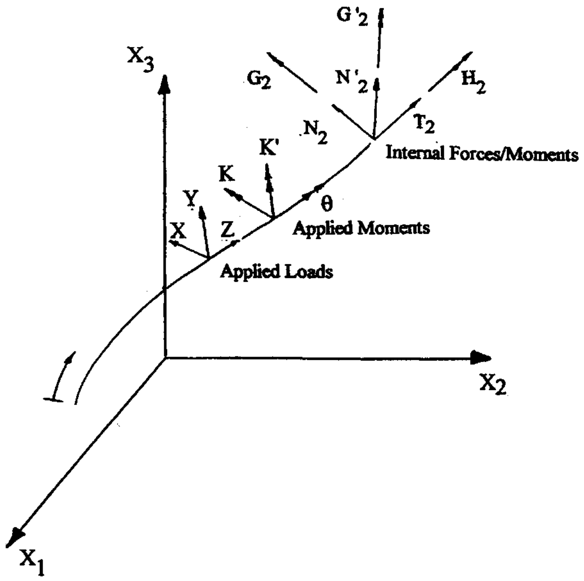

Fig. 2 Loads acting on a rope (Costello[2]).

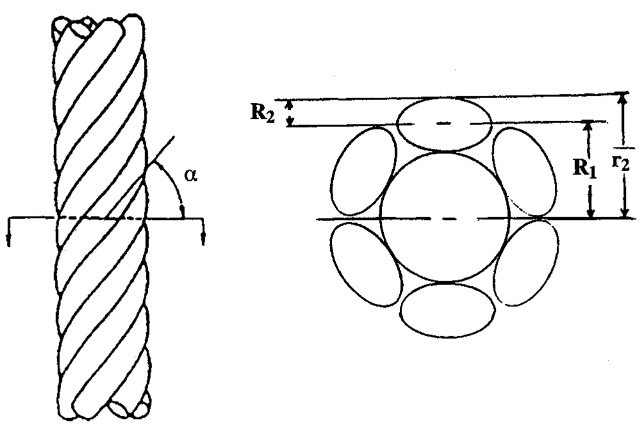

Fig. 3 Two-level hierarchy rope geometry (Costello[2]).

lieve that this concept is not always examined in the design of composites. The results presented here support the notion that multimaterial helically wound fibers add a rich variety of response mechanisms that can be manipulated by designers.

Geometrical Considerations

Costello[2] and Lee[3] have made significant contributions to the study of ropes. Figure 1 is included for the sake of completeness and to illustrate the nature of hierarchy. The geometry of these structural elements is complex and influences the extent to which mechanical damage is tolerated. A single untwisted wire is helically wound around a core, and when several wires are included, the resulting component is a strand. Several such strands lead to a rope, and the arrangement of these strands may vary and lead to a variety of architectures of ropes. The geometrical and kinematical relationships have been clearly established by Costello[2] based on the classical considerations of differential geometry of a particle moving on a space curve. These considerations lead to calculations of curvatures and twist that feature in the governing differential equations. Figure 2 from Costello[2] shows the principal parameters where X, Y, and Z are forces applied/unit length; K, K', and θ are moments applied/unit length; T (axial load), N, N' (shears), G, G', and H (moments) are the internal forces and moments and are the dependent variables; and k, k', and τ are the curvatures and twists and serve as parameters. When the space curve is used as an element of a strand, the helical angle, the number of wires around a core, and their geometry serve as additional parameters (see Fig. 3). The analysis proceeds on the assumption that the outer wires do not touch each other, and the mathematical condition satisfying that requirement is derived by Costello.[2] The basic relations needed to obtain the numerical results are summarized in the Appendix.

Discussion of Results

The scope of this limited numerical analysis was restricted to calculating the structural characteristics, under tensile load, of a strand made of two materials: graphite for the core and tungsten for the outer fibers. The strength characteristics of these materials are such that the inner fiber breaks first and the helical outer fibers continue to stretch to carry the applied load until failure.

A typical characteristic response, in a strain controlled-type of test condition, is shown in Fig. 4. The variable $\bar{a}$ is the ratio of the radius of the outer wire to that of the inner wire; $\bar{a}$ = 0.408 represents a condition in which the area of the inner wire is equal to the sum of the areas of the outer wires. Effective stress (using the total areas) is plotted as a function of percent strain with the helical angle as a parameter. The inner fiber breaks first because its ultimate strength is reached first as indicated by the sudden drop in Fig. 4. Further increase in strain is now resisted entirely by the outer helical fibers. The modulus for the second phase is less than that for the entire strand. As an example, normalized toughness (area under the stress-strain curve) is plotted in Fig. 5, the normalization being with respect to toughness corresponding to helical angle of 90 deg. Toughness is higher for all values of α as long as $\bar{a}$ is 0.3 or lower (i.e., decreasing diameter of outer fibers). As $\bar{a}$ increases (0.4 or higher), there is in fact a loss in toughness. The abrupt breaks in the curves indicate the angles where the outer wires' contact condition is violated.

For the case when both the outer and inner fibers are made of the same material, the inner wire diameter can be chosen such that

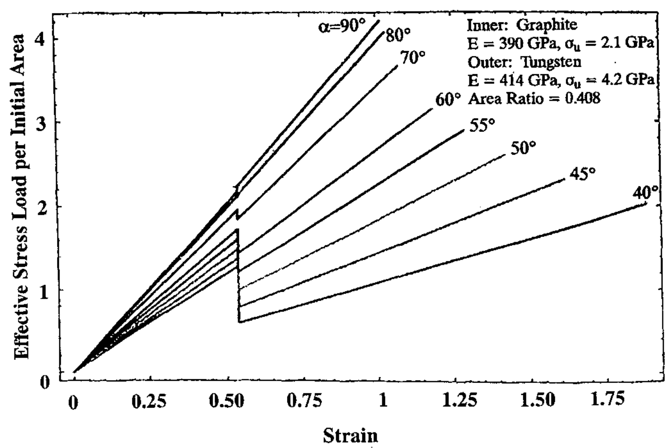

Fig. 4 Effective stress-strain response.

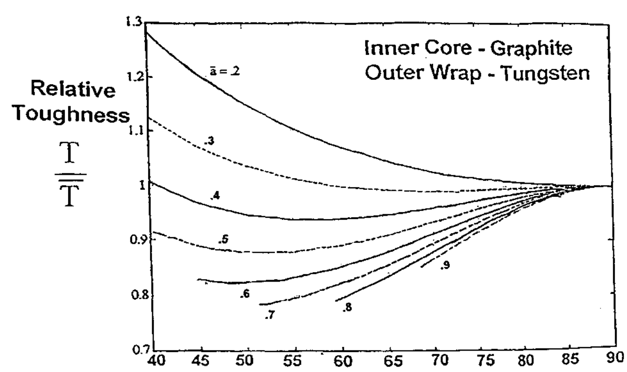

Fig. 5 Relative toughness.

it will break first. The advantage of the architecture is evident in that the ductility increases with a corresponding increase in toughness, but there is clearly a loss in stiffness for the strand.

Conclusions

This preliminary study shows the advantages of the strand in increased toughness at the expense of loss of stiffness, a characteristic quite typical of biological structures. The "design" of the latter is such that any increase in toughness is at the cost of some reduction in stiffness. Whether man-made structures can adapt this principle is an open question but is worth further examination. In so far as use of such strandlike or ropelike fibers in composites is concerned, we continue to look into the question of behavior in compression to examine buckling and bird-caging problems. Even when those issues are resolved satisfactorily, there is of course the larger question of manufacturability. Nevertheless, the concept appears attractive and deserves further scrutiny by interested researchers.

Appendix: Governing Equations[2]

Curvatures:

$\varkappa$ = curvature with respect to x axis

$\varkappa'$ = curvatures with respect to y axis

τ = curvature with respect to z axis (twist per unit length)

Forces:

N_2 = shear force in the x direction

N_2' = shear force in the y direction

T_2 = axial force in the z direction

Moments:

G_2 = bending moment in the x direction

G_2' = bending moment in the y direction

H_2 = twisting moment in the z direction

Change in curvature:

$$\Delta\varkappa_2 = \bar{\varkappa}_2 - \varkappa_2 = 0$$

$$R_2\Delta\varkappa'_2 = R_2(\bar{\varkappa}'_2 - \varkappa'_2) = -\frac{2\sin\alpha_2\cos\alpha_2}{r_2/R_2}\Delta\alpha_2 + \nu\frac{(R_1\xi_1 + R_2\xi_2)}{r_2}\frac{\cos^2\alpha_2}{r_2/R_2}$$

$$R_2\Delta\tau_2 = R_2(\bar{\tau}_2 - \tau_2) = +\frac{(1-2\sin^2\alpha_2)}{r_2/R_2}\Delta\alpha_2 + \nu\frac{(R_1\xi_1 + R_2\xi_2)}{r_2}\frac{\sin\alpha_2\cos\alpha_2}{r_2/R_2}$$

where the barred quantities represent the state after deformation.

Normalized forces:

$$\frac{G'_2}{ER_2^3} = \frac{\pi}{4}R_2\Delta\varkappa'_2, \qquad \frac{H_2}{ER_2^3} = \frac{\pi}{4(1+\nu)}R_2\Delta\tau_2, \qquad \frac{T_2}{ER_2^2} = \pi\xi_2$$

$$\frac{N'_2}{ER_2^2} = +\frac{H_2}{ER_2^3}\frac{\cos^2\alpha_2}{r_2/R_2} - \frac{G'_2}{ER_2^3}\frac{\sin\alpha_2\cos\alpha_2}{r_2/R_2}$$

Total force on the strand:

$$F_t = F_1 + F_2, \qquad M_t = M_1 + M_2, \qquad \frac{F_1}{ER_1^2} = \pi\xi_1$$

$$\frac{M_1}{ER_1^3} = \frac{\pi}{4(1+\nu)}R_2\tau_3$$

$$\frac{F_2}{ER_2^2} = m_2\left[\frac{T_2}{ER_2^2}\sin\alpha_2 + \frac{N'_2}{ER_2^2}\cos\alpha_2\right]$$

$$\frac{M_2}{ER_2^3} = m_2\left[\frac{H_2}{ER_2^3}\sin\alpha_2 + \frac{G'_2}{ER_2^3}\cos\alpha_2 + \frac{T_2}{ER_2^2}\frac{r_2}{R_2}\cos\alpha_2 - \frac{N'_2}{ER_2^2}\frac{r_2}{R_2}\cos\alpha_2\right]$$

where m_2 = number of outer wires. Note: subscript 2 refers to outer wires.

Stress in inner wire:

Normal stress due to $F_1' = F_1/\pi R_1^2$

Shear stress due to $M_1' = 2M_1/\pi R_1^3$

Stress in outer wire:

Normal stress due to $T_2' = T_2/\pi R_2^2$

Normal stress due to $G_2' = 4G_2/\pi R_2^3$

Shear stress due to $H_2' = 2H_2/\pi R_2^3$

Acknowledgment

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References

[1]Srinivasan, A. V., Haritas, G., and Hedberg, F. L., "Biomimetics: Advancing Man-Made Materials Through Guidance From Nature," *Applied Mechanics Reviews*, Nov. 1991, Chaps. 2 and 3.

[2]Costello, G. A., *Theory of Wire Rope*, Springer-Verlag, New York, 1991.

[3]Lee, W. K., "An Insight into Wire Rope Geometry," *International Journal of Solids and Structures*, Vol. 28, No. 4, 1991, pp. 471–490.

Some Nonlinear Response Characteristics of Collapsing Composite Shells

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Introduction

An interesting feature observed in the nonlinear numerical dynamic analysis of shells has been the appearance of chaos-like phenomena in postcollapse. Many physical systems exhibit chaotic vibrations, including the vibrations of buckled elastic structures.[1] For the purposes of definition, precollapse cases are those in which the applied load, in this case a sudden load of infinite duration, does not produce collapse; the result of the load is an oscillation of the shell structure about a deformed geometry that would have resulted had the same load been applied statically. Post-collapse refers to dynamic collapse or snap-through response. The correct choice of time step Δt is essential in nonlinear dynamic

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